Guided Search for Hybrid Systems

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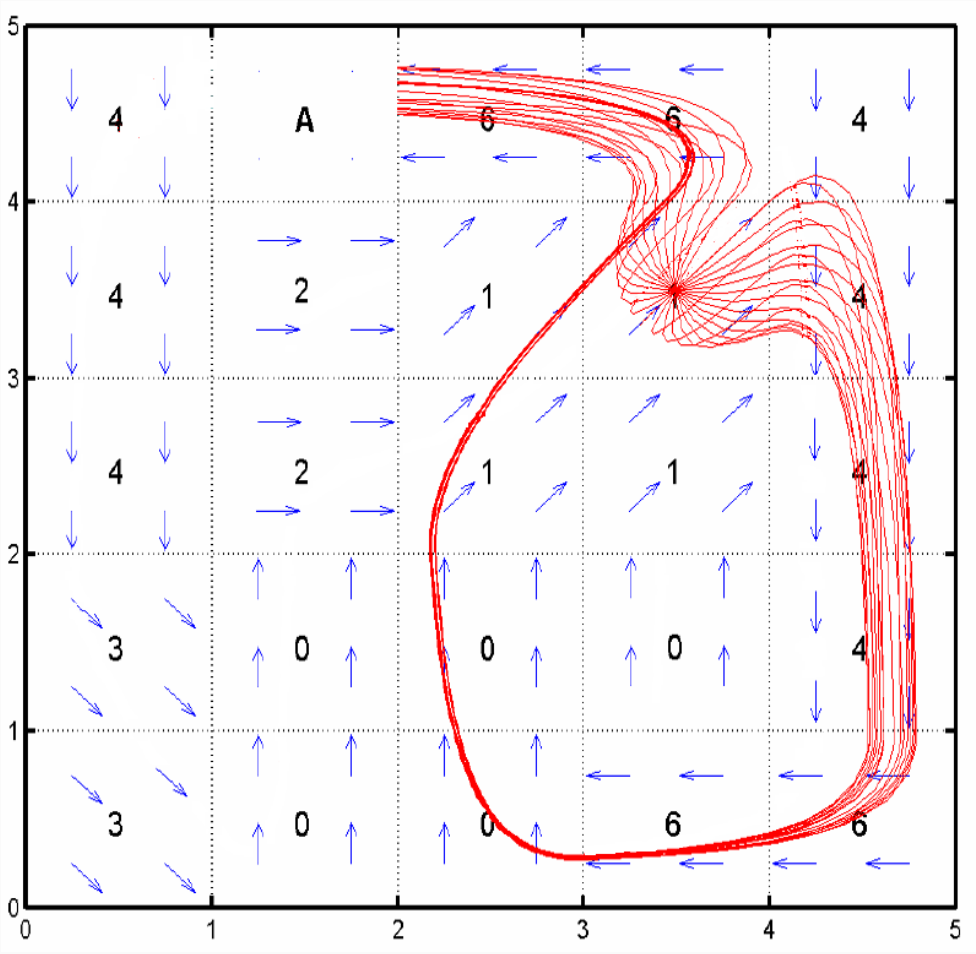
NAV Example



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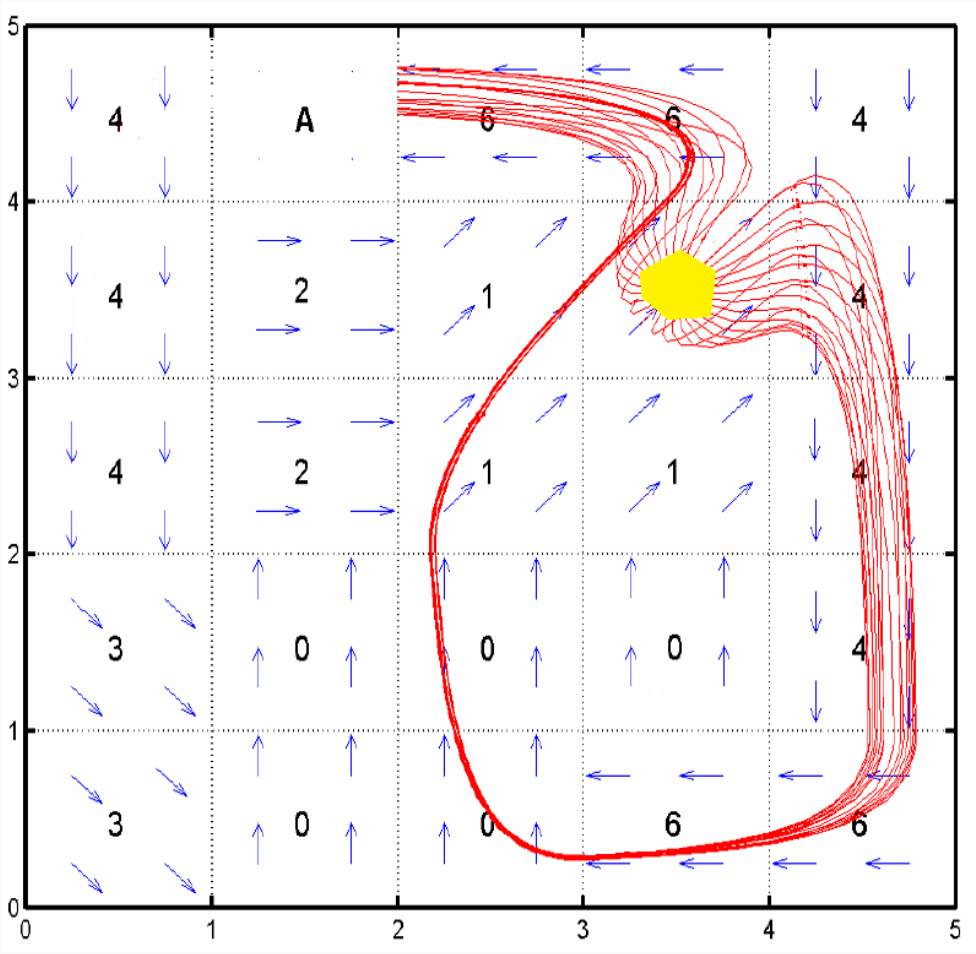

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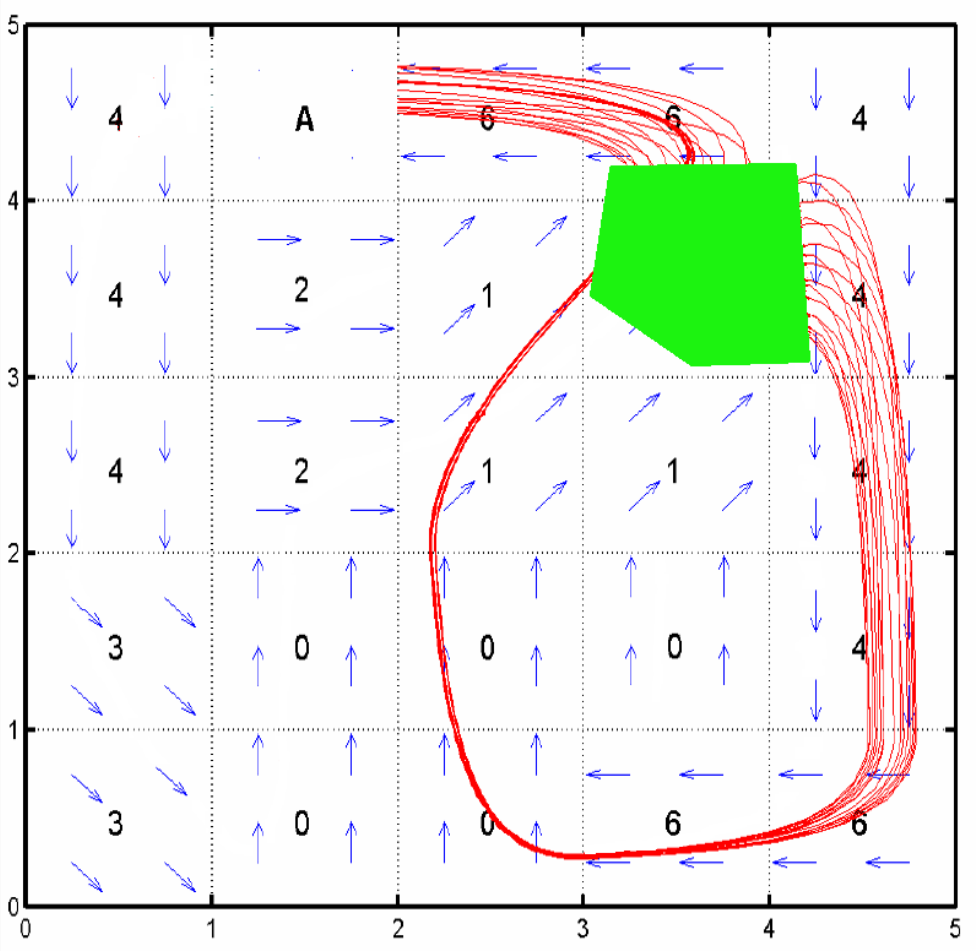

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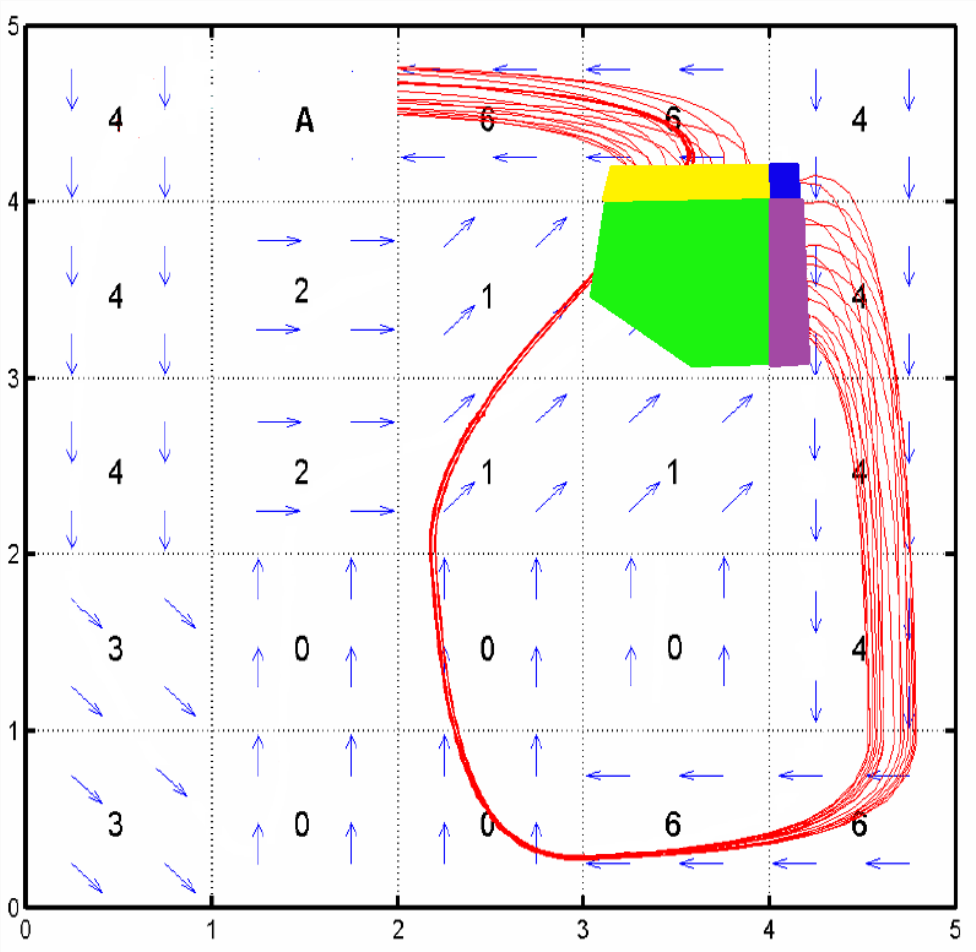


Falsification for Hybrid Systems (1)

- **Goal:** Find errors as fast as possible
- **Problem:** Hybrid MC algorithms not scalable
- **Approach:** Heuristics to guide symbolic search (up to 700 times performance improvement)
- **Generic:** Heuristics are computed automatically based on abstractions
- **Question:** Appropriate abstractions for this purpose?

Falsification for Hybrid Systems (2)

- **Current abstraction application: verification**
- **Novel abstraction application:**
  - **falsification**
  - **more precise abstractions – better search behavior**
  - **algorithm stays sound for every abstraction**
- **Scope: Related to AI planning (search not for a bad state, but for a good state)**

Outline

- **Box based distance approach**
  - **Focuses on continuous dynamics**
- **Pattern-database approach**
  - **Focuses on both discrete and continuous behavior**

Box-based distance approach

- **Goal: First consider paths leading to bad states which will probably require less resources to be computed**
- **Observation: Continuous post computation is the most time consuming part in the reachability analysis of hybrid systems**
- **Approximation: Minimize accumulated dwell time (i.e., passed time) in the case of hybrid systems**

Accumulated dwell time

- **Define cost(s) as a minimal sum of dwell times ranging over the error traces that start in s**
- **Problem: Exact computation of cost(s) is computationally infeasible**
- **Solution: Find a function h(s) which approximates cost(s) and still posesses enough precision to provide clever guidance**

Order-Preserving

- **A cost measure h is order-preserving with respect to cost if cost(s) < cost(s') implies h(s) < h(s')**
- **Observation: Order-preserving cost measures allow perfect prioritization**

Trajectory-Based Distance Measure

$dist_E(s)$:= length of a shortest trajectory from s to an error state.

Restricted Hybrid Systems

Hybrid system is called restricted if

- **all differential equations in H are of the form $\dot{x}_i(t) = \pm c_i$**

for every continous variable $x_i \in Var$ and a constant $c_i \in N$,

- **all updates in H are identity relations.**

Proposition: $dist_E(s)$ is order-preserving for restricted hybrid systems.

Euclidean distance measure (1)

- **Problem:** $dist_E(s)$ **cannot still be easily computed**
- **Solution: Instead of computing the exact length of trajectories between two points x and x′, use Euclidian distance measure** $dist_{eq}(x, x')$

Euclidean distance measure (2)

Observation: Euclidean distance $dist_{eq}(x, x')$ is not order-preserving for restricted systems.

Proposition: For restricted systems H with $\dot{x}_i(t) = c_i$, i.e. for restricted systems where all locations have the same continuous behavior, $dist_E^{eq}(x, x')$ is order-preserving.

Euclidean distance measure (3)

- **Coarse approximation of trajectory based measure**
- **Accurate when system dynamics only depends on the continuous state, e.g., when complex dynamics is approximated with a simpler one through state space partitioning:**
  - **phase portrait approximations (Henzinger et al. ’96)**
  - **approximation techniques employed by PHAVer (Frehse ’05)**
  - **hybridization techniques (Asarin et al. ’07)**
- **There exists a class of systems where it is even order-preserving**
- **Suggestion: Use Euclidean distance as a heuristic for general classes of hybrid systems**

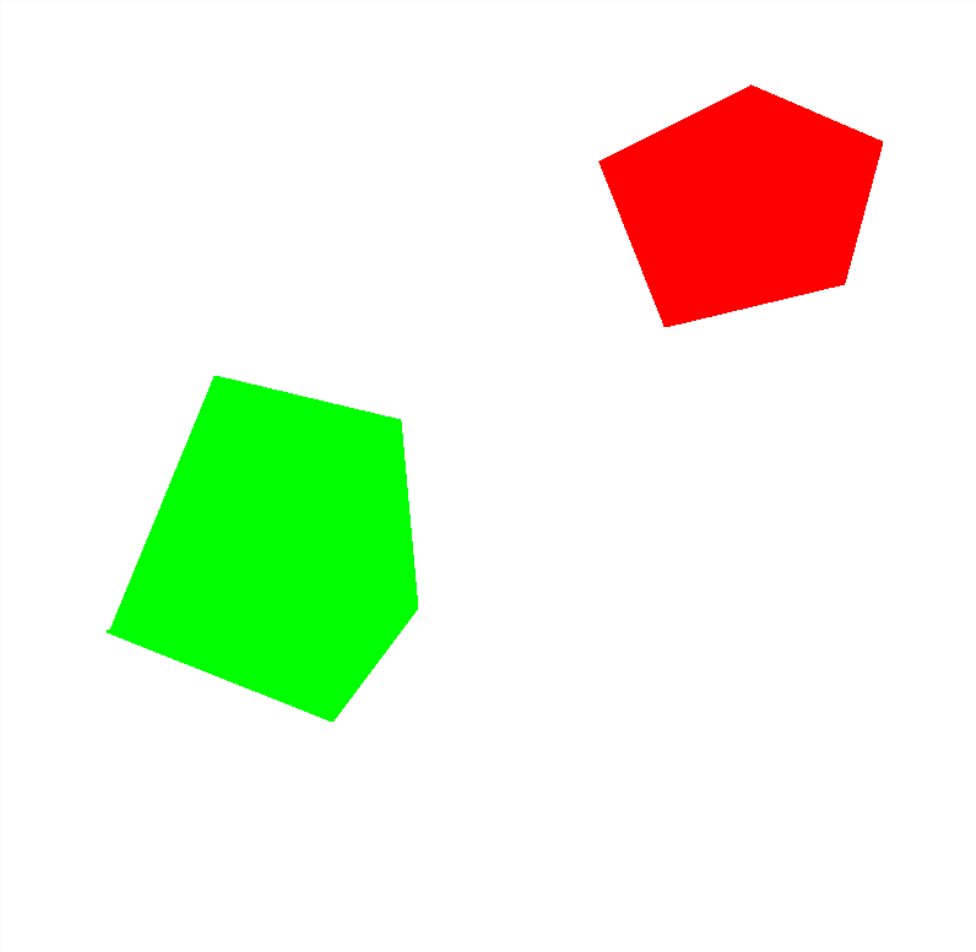
Box-based distance measure (1)

Function $h^{eq}(s)$ is called box-based distance measure if for every $s = (\ell, R)$ it returns the length of the shortest segment connecting the centers of the smallest bounding boxes of region R and some bad region.

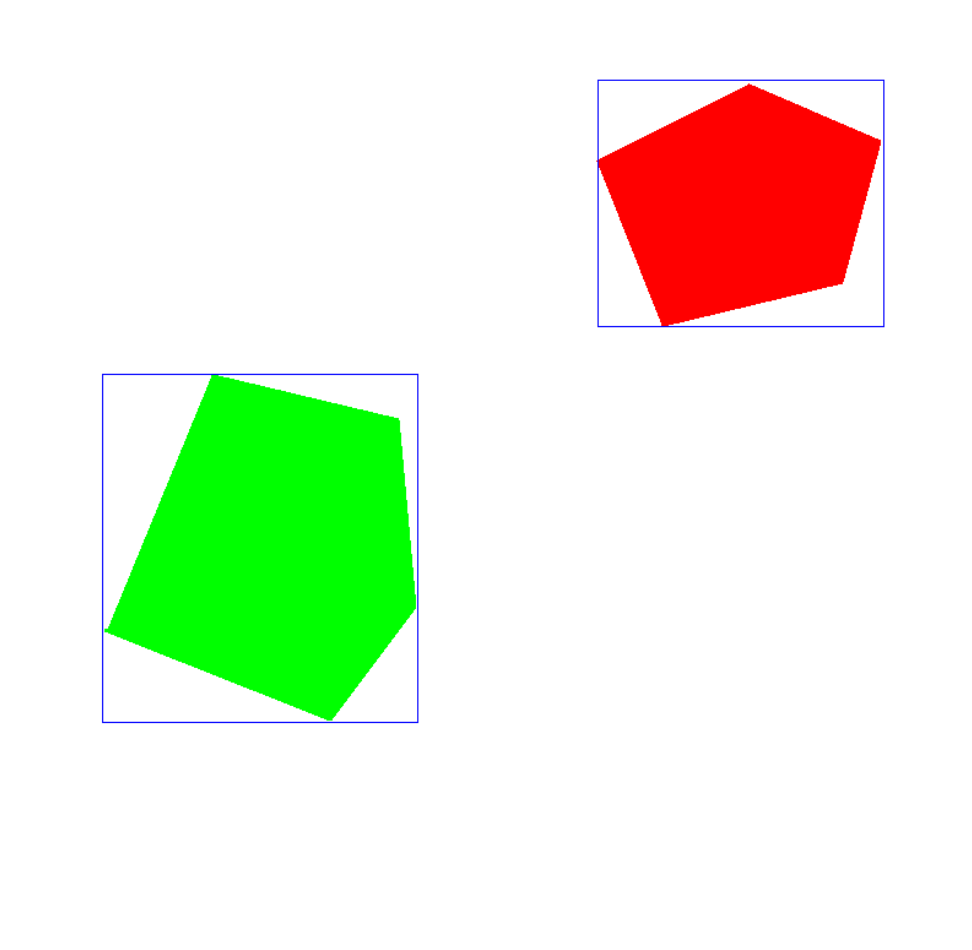
Formally,

$$h^{eq}(s) := min_{s_e=(\ell_e,R_e)} dist_{eq}(Center(\mathrm{B}(R)), Center(\mathrm{B}(R_e)))$$

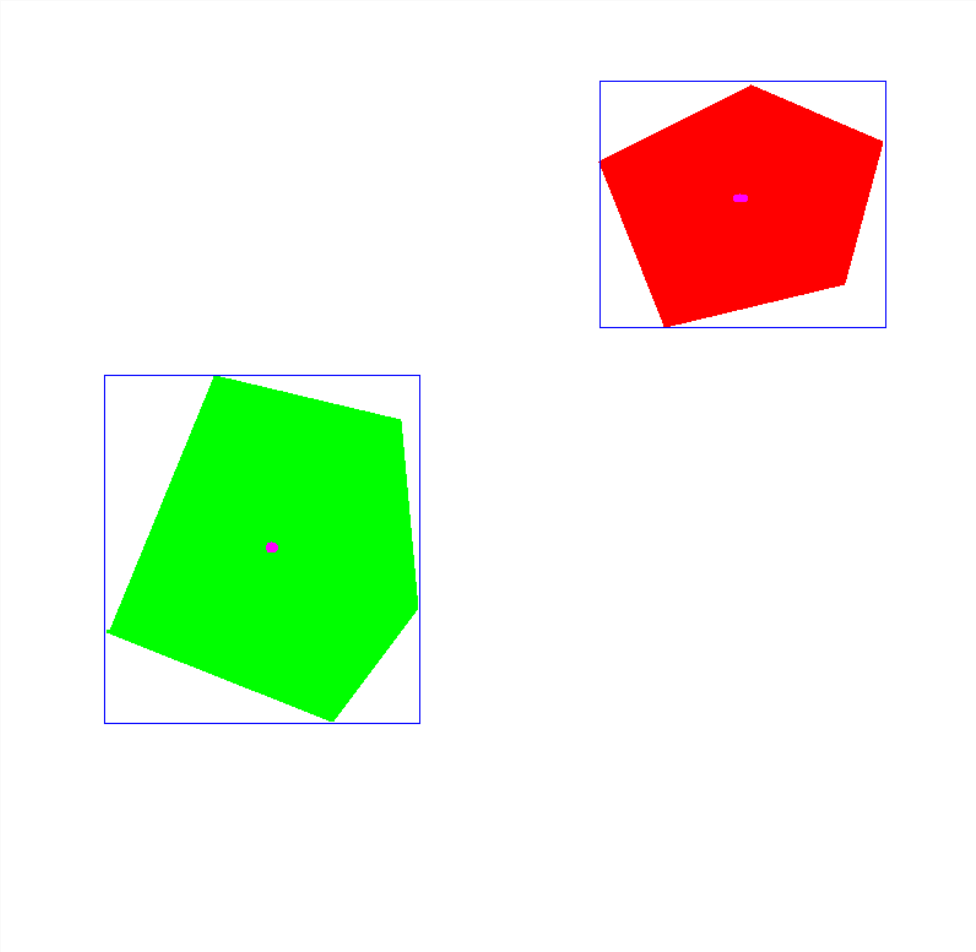
Box-based distance measure (2)



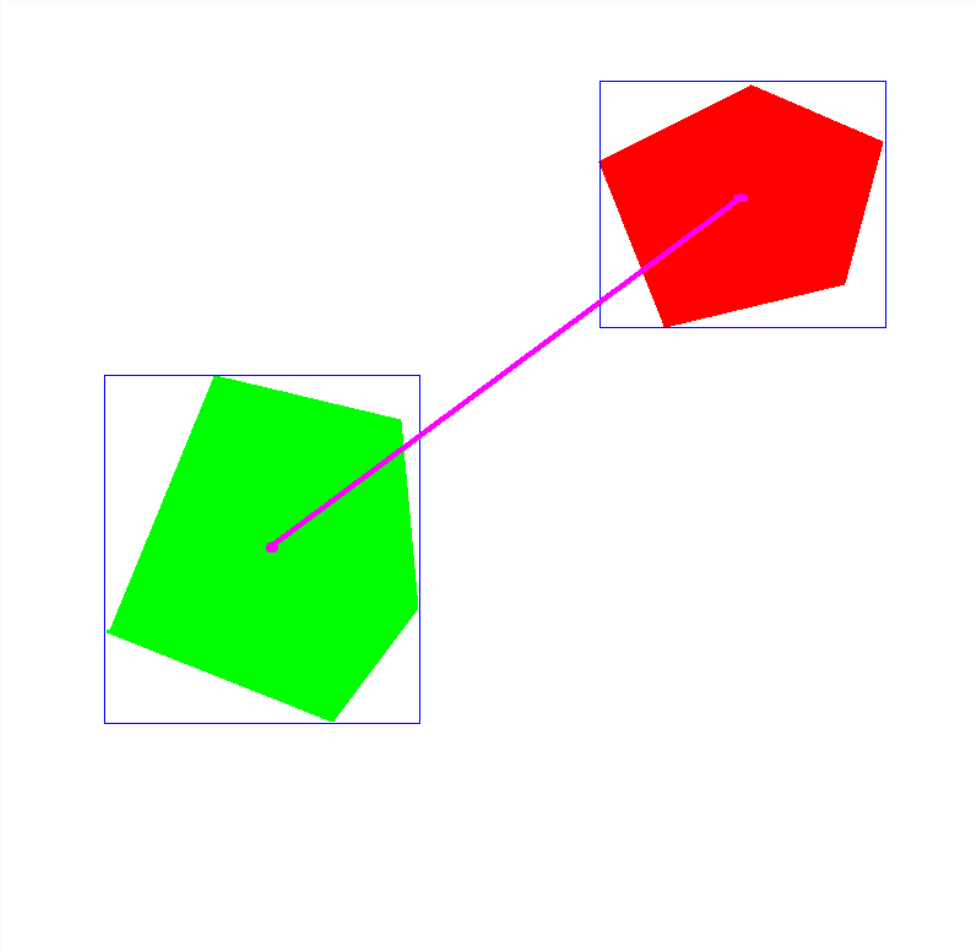
Box-based distance measure (3)



Box-based distance measure (4)



Box-based distance measure (5)



Experiments

- **Our techniques are implemented in SpaceEx model checker**
- **Models: Satellites on the orbit and navigation benchmark**
- **High degree of non-determinism and large branching factor**

Satellite Benchmark

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2	36	464	24	**101.3**	473	13	116.9
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13	576	n/a	n/a	OOM	579	52	**579.7**
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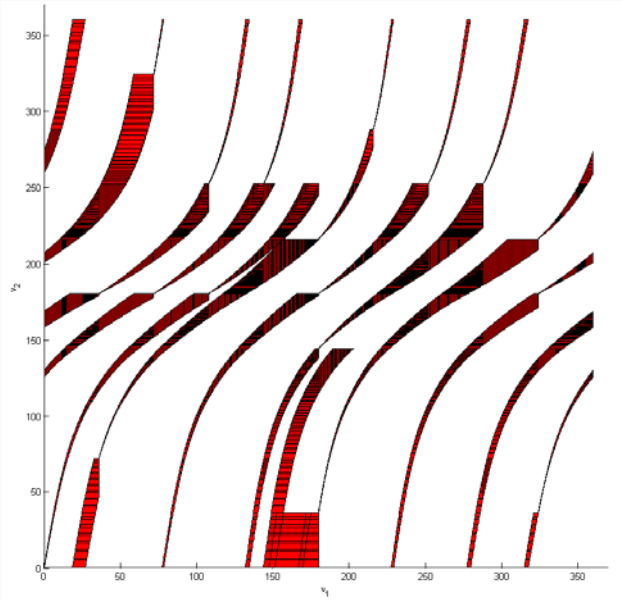
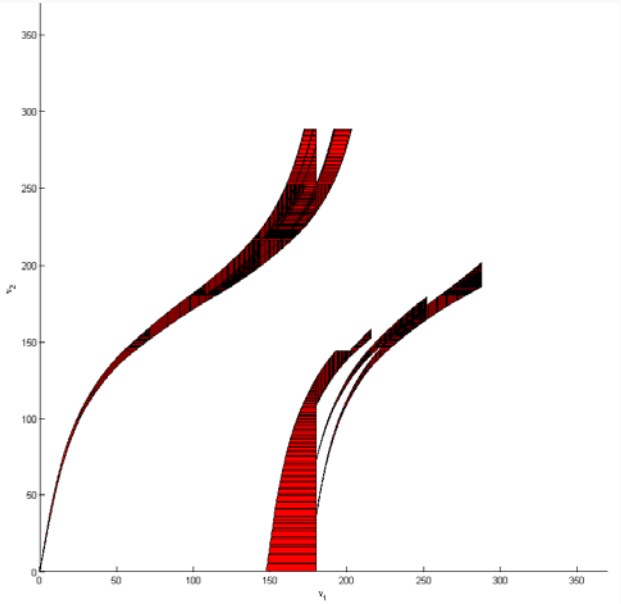




Uninformed DFS

Box-heuristic

Navigation Benchmark

Inst.	#loc	Uninformed DFS			Box heuristic		
		#it	length	time	#it	length	time
1	400	122	15	145.8	62	15	**70.5**
2	400	183	33	186.9	86	33	**120.4**
3	625	75	33	70.7	34	33	**36.6**
4	625	268	158	261.9	231	158	**209.6**
5	625	85	79	118.8	26	25	**37.8**
6	625	96	53	110.8	101	53	**104.9**
7	625	227	34	198.9	105	34	**96.9**
8	625	178	25	266.1	86	25	**137.3**
9	625	297	17	356,0	102	17	**131.9**
10	625	440	30	534,0	136	30	**201.8**
11	900	234	72	269.3	129	21	**149.1**
12	900	317	43	339.1	174	61	**198.3**
13	900	367	37	421.9	148	37	**190.4**
14	900	411	32	434.6	278	32	**297.9**
15	900	379	44	445.9	107	44	**137.8**

Outline

- Box based distance approach
  - Focuses on continuous dynamics
- **Pattern-database approach**
  - **Focuses on both discrete and continuous dynamics**

PDB approach

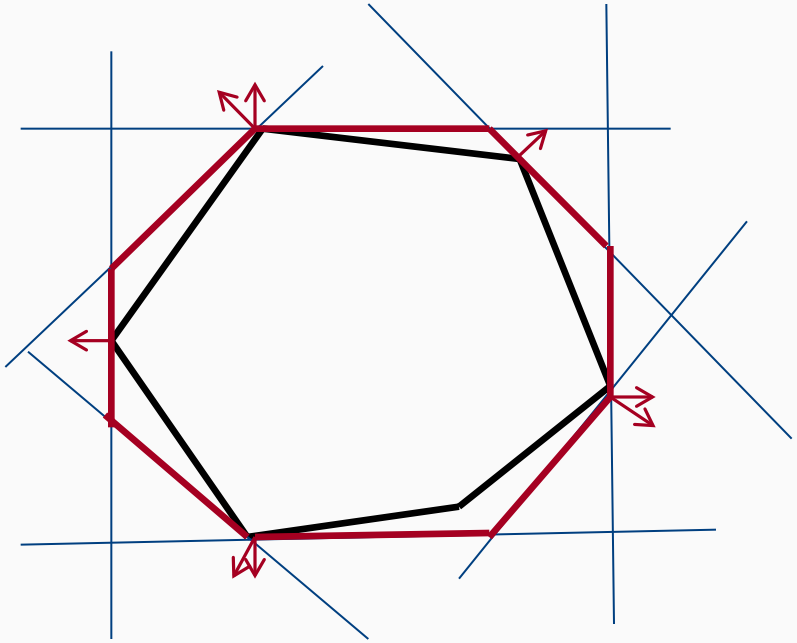
- **Goal:** Develop heuristic which
  - **Efficiently:** take discrete structure into account
  - **Utilize:** pattern database (PDB) approach
- **Precomputation:** Explore abstract state space
  - **Reachable state:** Compute its distance to the bad state
- **Analysis:** Explore concrete state space
  - **Use:** Abstract distance to select successors

Abstraction

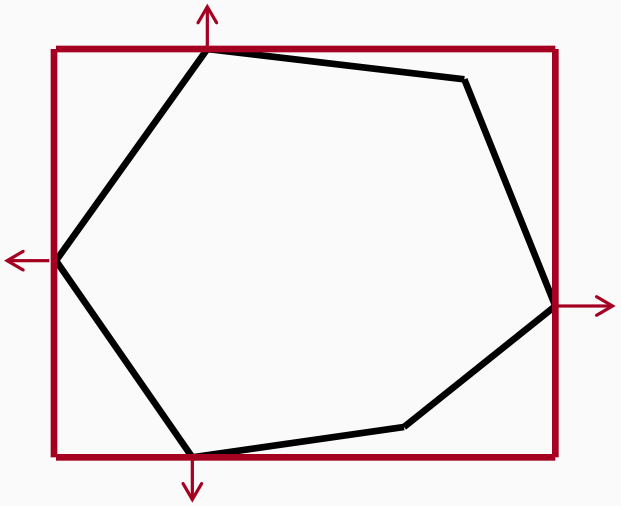
- **Utilize** the internal representation of polyhedra
  - **Based on:** Support functions implemented in SpaceEx
- **Precision** of polyhedra representation depends
  - **Direction set:** Direction and No of polyhedral faces
  - **Sampling time:** Time increment of continuous part

Abstraction: Direction Sets

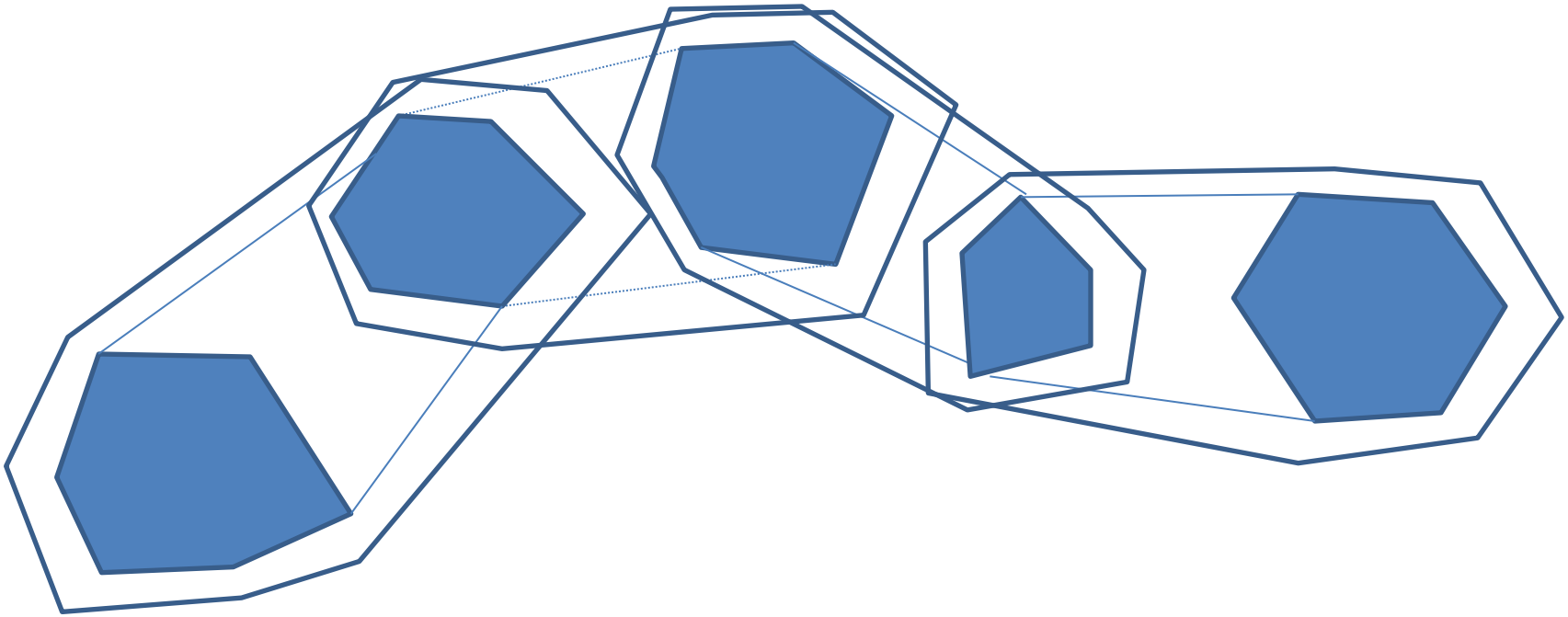
Octagonal directions (45 degrees)



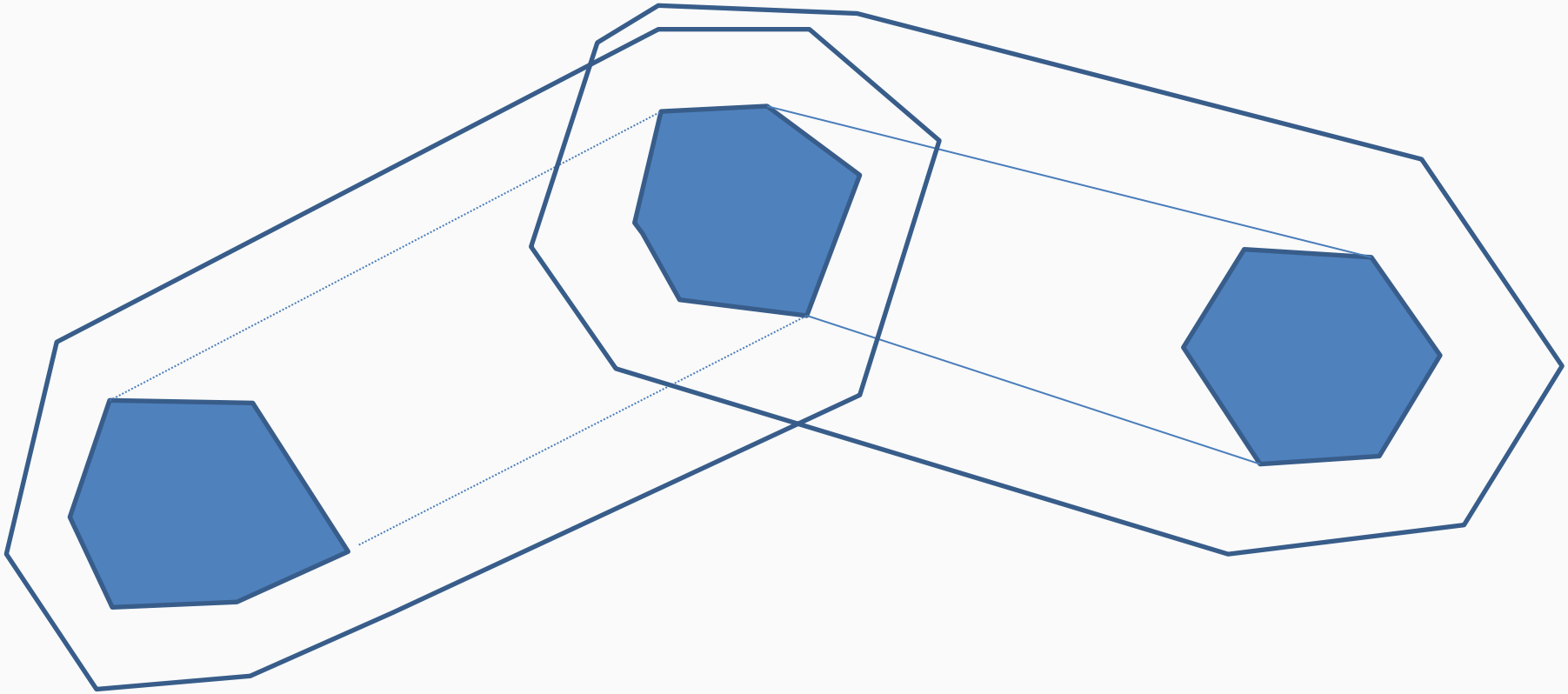
Box directions (90 degrees)



Abstraction: Sampling Time



Abstraction: Larger Sampling Time



Abstraction: Time-Directions

Thus by considering:

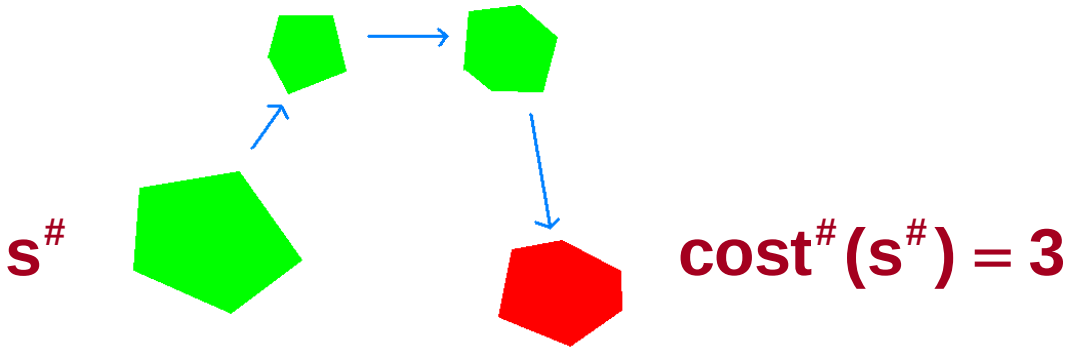
- **Smaller direction sets**
- **Larger sampling time**

we end up with coarser representation

Abstract Cost function

Given abstract symbolic state $s^{\#}$

the abstract cost function $cost^{\#}(s^{\#})$ is equal to the minimal number of discrete jumps to reach an abstract error state from $s^{\#}$



Concrete cost function

- Concrete cost function $cost^P(s) = cost^\#(s^\#)$
  - $s^\#$ is a corresponding abstract state to s
  - $cost^\#$ is an abstract cost funtion
- Abstract state $s^\#$ is called corresponding to s if
  - $s.discrete = s^\#.discrete$ same discrete part
  - $s.cont \subseteq s^\#.cont$ inclusion among cont parts
  - $cost^\#(s^\#)$ minimal among AS satisfying above

Partial Pattern Data Base (PDB)

- **Downside of complete PDB:**
  - **Precomputation might become quite expensive**
- **Solution is to consider partial PDB:**
  - **Explore only a fraction of the abstract region space (AS)**
  - **Focusing on AS likely to suffice for the concrete search**

Partial PDB

- **PDB which contains only**
  - **Abstract state/cost value pairs for abstract states**
  - **That are part of some trajectory of shortest length**
  - **From an initial state to an abstract error state**

$$\text{cost}^{PP}(s) = \begin{cases} \text{cost}^{\#}(s^{\#}), & \text{if there is a corresponding } s^{\#} \text{ to } s \\ \text{Infinity}, & \text{otherwise} \end{cases}$$

Partial PDB

- Utilize BFS and stop as soon as
  - All paths of minimal length to
  - An abstract bad state are explored
- Contains only abstract states
  - Explored on some shortest trajectory
- Equivalent to complete PDB behavior if
  - For every considered concrete state
  - A corresp abstract state in partial PDB can be found

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13	576	n/a	n/a	OOM	579	52	579.7	58	52	**163.2** (82.0)
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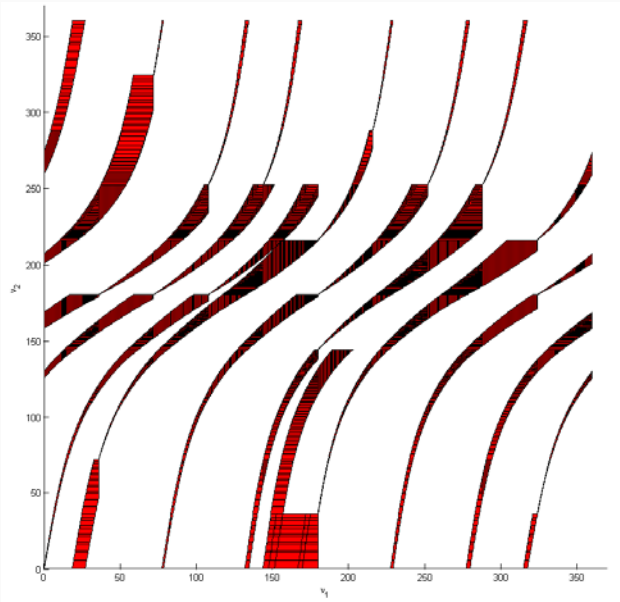
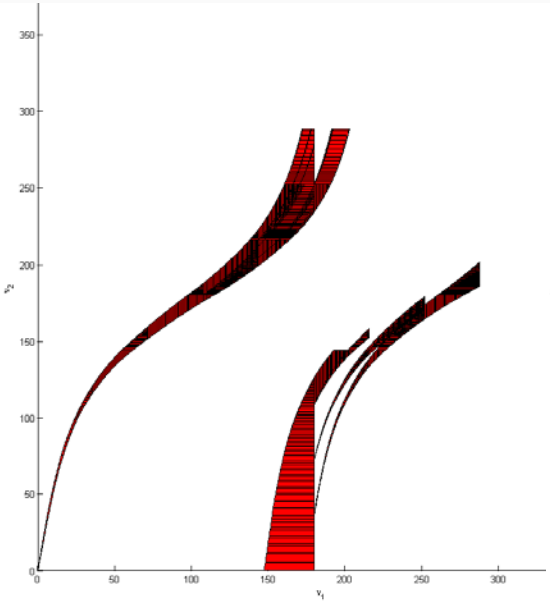
4	100	111	107	38.1	44	15	21.1	23	14	**14.8** (6.0)
5	100	109	104	262.9	45	15	178.6	23	14	**62.9** (5.9)
6	159	2170	∞	78.9	1352	∞	49.9	0	∞	**15.6** (15.6)
7	324	323	102	105.6	1289	106	457.7	25	24	**32.1** (8.8)
8	557	1637	42	45.8	936	42	**26.3**	156	42	44.1 (39.7)
9	574	7113	41	223.6	561	10	17.5	14	10	**6.6** (6.2)
10	575	9092	4	284.8	387	5	12.3	15	4	**2.4** (2.0)
11	576	5693	3769	816.6	257	13	36.5	15	13	**9.9** (5.9)
12	576	32966	13	7059.5	826	13	118.9	15	13	**10.0** (5.8)
13	576	n/a	n/a	OOM	579	52	579.7	58	52	**163.2** (82.0)
14	1293	13691	∞	436.1	7719	∞	249.6	0	∞	**135.5** (135.5)
15	1293	n/a	n/a	OOM	1806	142	1869.7	206	139	**617.4** (434.7)

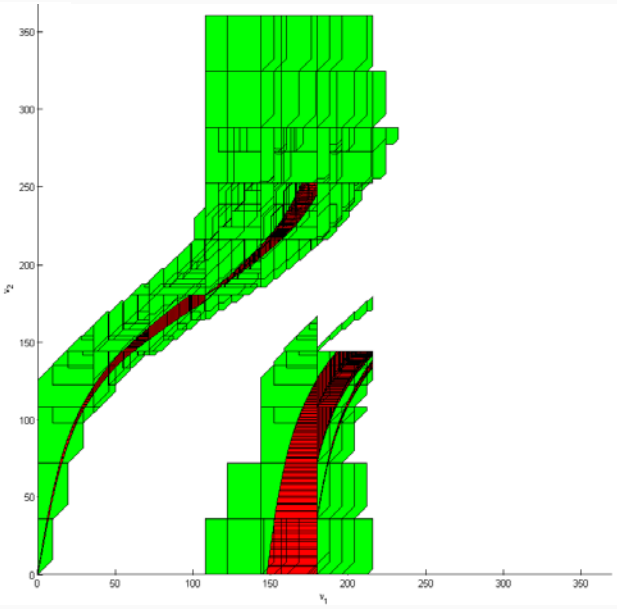
Satellite Benchmark

Inst.	#loc	Uninformed DFS			Box heuristic			PDB		
		#it	length	time	#it	length	time	#it	length	time
1	36	116	32	27.1	75	10	13.4	16	10	**10.3** (7.4)
2	36	464	24	101.3	473	13	116.9	30	13	**16.3** (12.2)
3	64	718	87	31.5	278	87	**11.0**	263	121	20.4 (9.5)
4	100	111	107	38.1	44	15	21.1	23	14	**14.8** (6.0)
5	100	109	104	262.9	45	15	178.6	23	14	**62.9** (5.9)
6	159	2170	∞	78.9	1352	∞	49.9	0	∞	**15.6** (15.6)
7	324	323	102	105.6	1289	106	457.7	25	24	**32.1** (8.8)
8	557	1637	42	45.8	936	42	**26.3**	156	42	44.1 (39.7)
9	574	7113	41	223.6	561	10	17.5	14	10	**6.6** (6.2)
10	575	9092	4	284.8	387	5	12.3	15	4	**2.4** (2.0)
11	576	5693	3769	816.6	257	13	36.5	15	13	**9.9** (5.9)
12	576	32966	13	7059.5	826	13	118.9	15	13	**10.0** (5.8)
13	576	n/a	n/a	OOM	579	52	579.7	58	52	**163.2** (82.0)
14	1293	13691	∞	436.1	7719	∞	249.6	0	∞	**135.5** (135.5)
15	1293	n/a	n/a	OOM	1806	142	1869.7	206	139	**617.4** (434.7)

Satellite Benchmark







Uninformed DFS **Box-heuristic** **PDB**

Navigation Benchmark

Inst.	#loc	Uninformed DFS			Box heuristic			PDB		
		#it	length	time	#it	length	time	#it	length	time
1	400	122	15	145.8	62	15	70.5	16	15	**20.0** (1.9)
2	400	183	33	186.9	86	33	120.4	34	33	**53.9** (7.6)
3	625	75	33	70.7	34	33	**36.6**	34	33	44.7 (7.5)
4	625	268	158	261.9	231	158	209.6	159	158	**127.5** (10.5)
5	625	85	79	118.8	26	25	**37.8**	26	25	42.1 (3.7)
6	625	96	53	110.8	101	53	104.9	54	53	**76.3** (9.8)
7	625	227	34	198.9	105	34	96.9	35	34	**47.6** (9.4)
8	625	178	25	266.1	86	25	137.3	26	25	**43.5** (7.0)
9	625	297	17	356,0	102	17	131.9	18	17	**30.8** (7.6)
10	625	440	30	534,0	136	30	201.8	31	30	**60.9** (13.6)
11	900	234	72	269.3	129	21	149.1	22	21	**32.7** (8.1)
12	900	317	43	339.1	174	61	198.3	44	43	**62.8** (15.8)
13	900	367	37	421.9	148	37	190.4	38	37	**70.7** (20.1)
14	900	411	32	434.6	278	32	297.9	33	32	**57.7** (10.9)
15	900	379	44	445.9	107	44	137.8	45	44	**69.9** (9.0)

Conclusions

- **Done:** Novel heuristics for effective search guidance in concrete symbolic state space
- **Box-based heuristic:**
  - Light-weight on-the-fly heuristic estimation
  - Suited for systems with primarily continuous behavior
- **PDB heuristic:**
  - Involves pre-computation phase
  - Considers both discrete and continuous behavior
- **Future work:**
  - Combination of information through several heuristics
  - Exploration of complexity dimensions